Attendance Problems

1. Write a conditional from the sentence "An isosceles triangle has two congruent sides."

2. Write the contrapositive of the conditional "If it is Tuesday, then John has a piano lesson."

3. Show that the conjecture "If $x > 6$, then $2x > 14$" is false by finding a counterexample.

- I can write indirect proofs.
- I can apply inequalities in one triangle.

Vocabulary: Indirect proof

Common Core: CC.9-12.G.CO.10 Prove theorems about triangles.

So far you have written proofs using *direct reasoning*. You began with a true hypothesis and built a logical argument to show that a conclusion was true. In an **indirect proof**, you begin by assuming that the conclusion is false. Then you show that this assumption leads to a contradiction. This type of proof is also called a proof by contradiction.

Writing an Indirect Proof
1. Identify the conjecture to be proven.
2. Assume the opposite (the negation) of the conclusion is true.
3. Use direct reasoning to show that the assumption leads to a contradiction.
4. Conclude that since the assumption is false, the original conjecture must be true.

Q: What do you call a prisoner's poem?
A: A converse

"Be **excellent** to each other." -- Bill, in Bill and Ted's Excellent Adventure

Helpful Hint

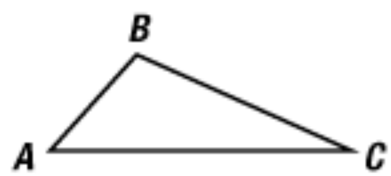
When writing an indirect proof, look for a contradiction of one of the following: the given information, a definition, a postulate, or a theorem.

Refer to video example 1.

Write an indirect proof that an obtuse triangle cannot be a right triangle.

Given: $\triangle ABC$ is obtuse.

Prove: $\triangle ABC$ is not a right triangle.



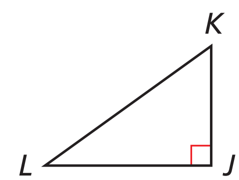
1 Writing an Indirect Proof

Write an indirect proof that a right triangle cannot have an obtuse angle.

Step 1 Identify the conjecture to be proven.

Given: $\triangle JKL$ is a right triangle.

Prove: $\triangle JKL$ does not have an obtuse angle.



Step 2 Assume the opposite of the conclusion.

Assume $\triangle JKL$ has an obtuse angle. Let $\angle K$ be obtuse.

Step 3 Use direct reasoning to lead to a contradiction.

$m\angle K + m\angle L = 90°$	*The acute ∠s of a rt. △ are comp.*
$m\angle K = 90° - m\angle L$	*Subtr. Prop. of =*
$m\angle K > 90°$	*Def. of obtuse ∠*
$90° - m\angle L > 90°$	*Substitute 90° − m∠L for m∠K.*
$m\angle L < 0°$	*Subtract 90° from both sides and solve for m∠L.*

However, by the Protractor Postulate, a triangle cannot have an angle with a measure less than 0°.

Step 4 Conclude that the original conjecture is true.

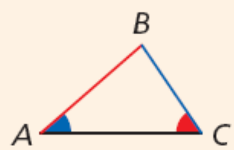
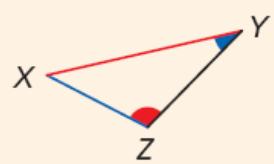
The assumption that $\triangle JKL$ has an obtuse angle is false.

Therefore $\triangle JKL$ does not have an obtuse angle.

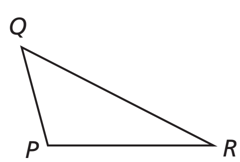
Example 1. Write an indirect proof that if $a > 0$, then $\frac{1}{a} > 0$.

4. Guided Practice. Write an indirect proof that a triangle cannot have two right angles.

Theorems — Angle-Side Relationships in Triangles

THEOREM	HYPOTHESIS	CONCLUSION
5-5-1 If two sides of a triangle are not congruent, then the larger angle is opposite the longer side. (In △, larger ∠ is opp. longer side.)	 $AB > BC$	$m\angle C > m\angle A$
5-5-2 If two angles of a triangle are not congruent, then the longer side is opposite the larger angle. (In △, longer side is opp. larger ∠.)	 $m\angle Z > m\angle Y$	$XY > XZ$

Theorem 5-5-2



Given: $m\angle P > m\angle R$
Prove: $QR > QP$

Indirect Proof:
Assume $QR \not> QP$. This means that either $QR < QP$ or $QR = QP$.

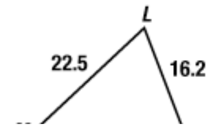
Case 1 If $QR < QP$, then $m\angle P < m\angle R$ because the larger angle is opposite the longer side. This contradicts the given information. So $QR \not< QP$.

Case 2 If $QR = QP$, then $m\angle P = m\angle R$ by the Isosceles Triangle Theorem. This also contradicts the given information, so $QR \neq QP$.

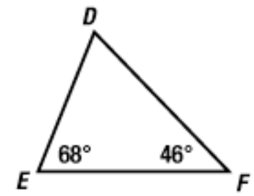
The assumption $QR \not> QP$ is false. Therefore $QR > QP$.

Refer to video example 2.

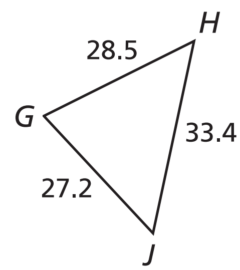
A. Write the angles in order from smallest to largest.



B. Write the side lengths from smallest to largest.



2 Ordering Triangle Side Lengths and Angle Measures

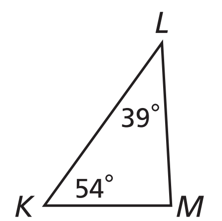


A Write the angles in order from smallest to largest.

The shortest side is $\overline{GJ}$, so the smallest angle is $\angle H$.

The longest side is $\overline{HJ}$, so the largest angle is $\angle G$.

The angles from smallest to largest are $\angle H$, $\angle J$, and $\angle G$.



B Write the sides in order from shortest to longest.

$m\angle M = 180° - (39° + 54°) = 87°$ *△ Sum Thm.*

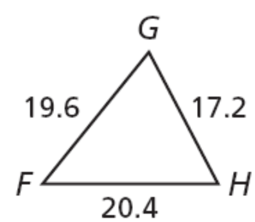
The smallest angle is $\angle L$, so the shortest side is $\overline{KM}$.

The largest angle is $\angle M$, so the longest side is $\overline{KL}$.

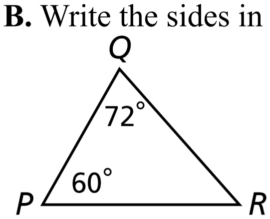
The sides from shortest to longest are $\overline{KM}$, $\overline{LM}$, and $\overline{KL}$.

Example 2.

A. Write the angles in order from smallest to largest.

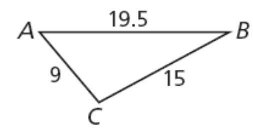


B. Write the sides in order from shortest to longest.

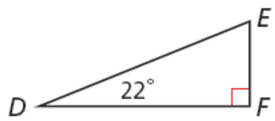


Guided Practice.

5. Write the angles in order from smallest to largest.



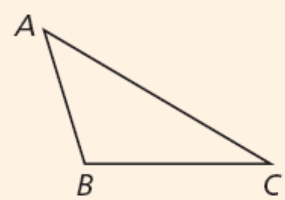
6. Write the sides in order from shortest to longest.



Theorem 5-5-3 **Triangle Inequality Theorem**

The sum of any two side lengths of a triangle is greater than the third side length.

$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$



Refer to video example 3. Tell whether a triangle can have the given side lengths.

A. 6, 10, & 14

B. 5, 9, & 14

C. a -1, $a^2 - 2$, and 3a when a = 4

3 Applying the Triangle Inequality Theorem

Tell whether a triangle can have sides with the given lengths. Explain.

A **3, 5, 7**

$3 + 5 \overset{?}{>} 7$	$3 + 7 \overset{?}{>} 5$	$5 + 7 \overset{?}{>} 3$
$8 > 7$ ✓	$10 > 5$ ✓	$12 > 3$ ✓

Yes—the sum of each pair of lengths is greater than the third length.

B **4, 6.5, 11**

$4 + 6.5 \overset{?}{>} 11$

$10.5 \not> 11$

No—by the Triangle Inequality Theorem, a triangle cannot have these side lengths.

C $\boldsymbol{n + 5, n^2, 2n}$**, when** $\boldsymbol{n = 3}$

Step 1 Evaluate each expression when $n = 3$.

$n + 5$	n^2	$2n$
$3 + 5$	3^2	$2(3)$
8	9	6

Step 2 Compare the lengths.

$8 + 9 \overset{?}{>} 6$	$8 + 6 \overset{?}{>} 9$	$9 + 6 \overset{?}{>} 8$
$17 > 6$ ✓	$14 > 9$ ✓	$15 > 8$ ✓

Yes—the sum of each pair of lengths is greater than the third length.

<u>Example 3</u>. Tell whether a triangle can have sides with the given lengths. Explain.

A. 7, 10, 19 **B.** 2.3, 3.1, 4.6

C. $n + 6$, $n^2 - 1$, $3n$, when $n = 4$.

<u>**Guided Practice**</u>**. Tell whether a triangle can have sides with the given lengths. Explain.**

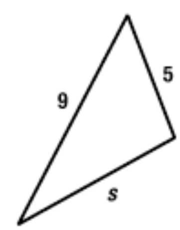
7. 8, 13, 21 **8.** 6.2, 7, 9

9. $t - 2$, $4t$, $t^2 + 1$, when $t = 4$.

5-5 Indirect proof and Inequalities in one triangle (pp 348) 16-19, 23, 25.

<u>**Refer to video example 4 on page 347.**</u>

The lengths of two sides of a triangle are 5 inches and 9 inches.
Find the range of possible lengths for the third side.



4 Finding Side Lengths

The lengths of two sides of a triangle are 6 centimeters and 11 centimeters. Find the range of possible lengths for the third side.

Let s represent the length of the third side. Then apply the Triangle Inequality Theorem.

$s + 6 > 11$	$s + 11 > 6$	$6 + 11 > s$
$s > 5$	$s > -5$	$17 > s$

Combine the inequalities. So $5 < s < 17$. The length of the third side is greater than 5 centimeters and less than 17 centimeters.

Example 4. The lengths of two sides of a triangle are 8 inches and 13 inches. Find the range of possible lengths for the third side.

10. Guided Practice. The lengths of two sides of a triangle are 22 inches and 17 inches. Find the range of possible lengths for the third side.

Refer to video example 5.

The distance from Austin to Mason is 108 miles, and the distance from Mason to San Antonio is 111 miles. What is the range of distances from Austin to San Antonio?

5 ***Travel Application***

The map shows the approximate distances from San Antonio to Mason and from San Antonio to Austin. What is the range of distances from Mason to Austin?

Let d be the distance from Mason to Austin.

$d + 111 > 78$	$d + 78 > 111$	$111 + 78 > d$	*△ Inequal. Thm.*
$d > -33$	$d > 33$	$189 > d$	*Subtr. Prop. of Inequal.*
	$33 < d < 189$		*Combine the inequalities.*

The distance from Mason to Austin is greater than 33 miles and less than 189 miles.

Example 5. The figure shows the approximate distances between cities in California. What is the range of distances from San Francisco to Oakland?

11. <u>Guided Practice</u>. The distance from San Marcos to Johnson City is 50 miles, and the distance from Seguin to San Marcos is 22 miles. What is the range of distances from Seguin to Johnson City?

5-5 Indirect proof and Inequalities in one triangle (pp 348) 16-19, 23, 25, 29, 31, 32, 48, 62.